Summation formulas and properties

Given a sequence $a_1, a_2, a_3 \cdots a_n$ of numbers, the finite sum $a_1+a_2+a_3+\cdots a_n$ where n is an nonnegative, can be written

$$\sum_{k=1}^{n} a_k$$

if $n=0$; the value of the summation is defined to be 0. The value of a finite series is always, and its term can be added in any order.

Given a sequence $a_1, a_2 \cdots$ of numbers, the finite sum $a_1+a_2+\cdots$ can be written

$$\boxed{\sum_{j=m}^{n} a_j = a_m + a_{m+1} + \cdots a_n}$$

ex:- $\sum_{j=1}^{3} j = 1+2+3 = 6$

* The variable j in this notation is called index of notation.

* Also, m and n are called the lower and upper limits of the summation.

Nested Summations:-

It is also common to nest summations within one another.

Q. What is the value of the following summation?

$$\sum_{i=1}^{2}\sum_{j=1}^{3} ij$$

Sol^n:-

$$\sum_{i=1}^{2}\sum_{j=1}^{2} ij = \sum_{i=1}^{2} i+2i+3i = \sum_{i=1}^{2} 6i$$

$$= 6+12=18$$

Q Bounding Summations:-

It is describe the running time of algorithms. Some methods are as follows:-

1. Mathematical Indoction:-

Q. What is mathematical Induction:- The technique that is used to proving the results or for natural numbers, the statements are established is known as mathematical Induction.

A mathematical technique used for providing a statement, formula or theorem is true for every natural number is known as Mathematical Induction.

A statement can be proved in two steps:-

Step 1:- (Base step):- The statement is proved to be true for the initial value.

Step 2:- (Inductive step):- The statemebt is true is considered. It is represented as an

$\boxed{n = \text{intial value}}$

How to do?

Step 1:- An initial value for which the statement is true considered. It is represented as

$n =$ initial

Step 2:- It is assumed that the statement is true for any value of $n=k$. Then the statement is proved true for $n=k+1$. is broken down into two parts, one part is $n=k$, which is the proved one and the second part is proved.

Q. $3^n - 1$ is multiple of 2 for $n = 1, 2 \ldots\ldots$

Soln:- $n=1$,

Step1: $3^1 - 1 = 2$, which is a multiple of 2.

Step 2:- It is assumed that $3^n - 1$ is true for $n=k$ and hence, $3^k - 1$ is true.

It is true to be proved that

$3^{k+1} - 1$ is also a multiple of 2.

1 is also a multiple of 2.

$$3^{k+1} - 1 = 3 \times 3^k - 1$$
$$= (2 \times 3^k) + (3^k - 1)$$

The first part $(3 \times 3k)$ is a multiple of 2 and the second part $(3k+1)$ is also true as our previous assumption.

Hence,

$\boxed{3^{k+1} - 1 \text{ is a multiple of } 2.}$

Bounding the terms :-

Use the largest (smallest) value of a term to bound others:-

$$a_1 + a_2 + \dots a_n \leq a_{max} + a_{max} + \dots + a_{max} = n \cdot a_{max}$$

$$a_1 + a_2 + \dots a_n \geq a_{min} + a_{min} + \dots + a_{min} = n \cdot a_{min}$$

ex:-

$$\sum_{k=1}^{n} k \quad \sum_{k=1}^{n} n = n \cdot \sum_{k=1}^{n} 1 = n^2 \Rightarrow \sum_{k=1}^{n} k = O(n^2)$$

$$\sum_{k=1}^{n} k \quad \sum_{k=1}^{n} 1 = n \Rightarrow \sum_{k=1}^{n} k = \Omega(n).$$